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ERROR ANALYSIS OF THE RECOVERY OF DATA ACQUIRED BY A VOLTAGE-TO-FREQUENCY DATA ACQUISITION SYSTEM FOR SINUSOIDAL, STEP, AND RAMP INPUTS

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NOTICES

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FOREWORD

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This technical report has been reviewed and is approved.

Edgar D. Smith Major, USAF AF Representative, LRF Directorate of Test	Roy R. Croy, Jr. Colonel, USAF Director of Test

ABSTRACT

The errors generated by the recovery of data acquired by a voltage-to-frequency (V/F) data acquisition system are theoretically analyzed, using a digital computer. The input waveforms considered are the sinusoid, the step function, and the ramp function. In each case, the well known "sampling theorem" of C. E. Shannon, viz, two samples per period of the smallest period present in the signal are sufficient for recovery of data with perfect fidelity, will be shown to be inapplicable as a criterion for accurate and reliable recovery of data acquired by a V/F system. The errors are of two categories: One is the error from linear interpolation recovery, and the other is from the fact that data points do not always coincide with the input. The sinusoid analysis reveals that a sampling rate of 11.4 summing intervals per period is required to assure 5 percent or less error. The step function analysis shows that the errors are independent of sampling rate; however, its response time is approximately equal to the summing interval. The analysis of the ramp function indicated that the ratio of ramp rise time to summing interval must be five or greater to assure 5 percent or less error. These criteria are unique to each case, and in the case of the sinusoid, are many times larger than the criterion of the "sampling theorem."

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TABLES

NOMENCLATURE

E Error at a data point

e Error at any point using linear interpolation

f	Frequency, Hz
Int ( )	Integer part of
K	Synchronization parameter for step and ramp input
K_v	Gain of V/F converter (pulses/sec-v)
m	Summing intervals per period of sinusoid
N	Output counts of V/F converter in τ seconds
n	Number of arbitrary summing interval
T	Period of sinusoidal input, sec
T_D	Time delay, sec
t	Time
V	Voltage
α	Argument of Eqs. (11) and (21)
β	Ratio of ramp rise time to summing interval
Δ ( )	Change in
μ_1	Step function
μ_2	Ramp function
τ	Length of summing interval, sec
ψ	Synchronization parameter for sinusoidal input

SUBSCRIPTS

i	Input
max	Maximum value
min	Minimum value
n	Value in summing interval number n
o	Output

SECTION I
INTRODUCTION

Data acquisition relies heavily on the process of sampling and representing a continuous time function in terms of discrete time samples. Numerous methods and systems to accomplish this are in existence and have their own uniquenesses. However, all of these methods and systems have a common basis that justifies the representation of a continuous variable by discrete samples, and that is the sampling theorem.

The sampling theorem, though known and used much earlier, is widely attributed to C. E. Shannon (Ref. 1). It may be stated as follows:

> "2/T samples per second suffice to represent perfectly and permit perfect recovery of a time function provided that the time function contains no periodic components of period less than T seconds."

However, the data recovery from an integrating system such as the voltage-to-frequency data acquisition system when used as a sampling system, i.e., used to recover the actual waveshape, does not have a direct application through the sampling theorem.

This investigation is a derivation of the number of integrating intervals required to accurately reconstruct an input using linear interpolation of the data points obtained from an integrating data acquisition system. Assuming that an error also exists at the data points, the analysis will include both a determination of data point error and linear interpolation error. Error will be the absolute value of the difference of the input and the representative output at a given time as a percentage of the maximum magnitude of the input. This is a modification of the definition of deviation as defined in Ref. 2, page 209.

Three ideal signals will be treated theoretically: sinusoid, step, and ramp. The digital computer will be used to simulate this type of data recovery, and the maximum error for various parameter configurations will be determined. From this, the limits of the parameters for accurate (5 percent) recovery can be determined.

Experimental verification of the error analysis for the sinusoid was performed using the VIDAR® voltage-to-frequency data acquisition system in the Large Rocket Facility at AEDC.

SECTION II
VOLTAGE-TO-FREQUENCY CONVERSION METHOD

The method of data acquisition that is to be treated in this study is a voltage-to-frequency (V/F) conversion. The block diagram in Fig. 1 depicts the setup of such a system.

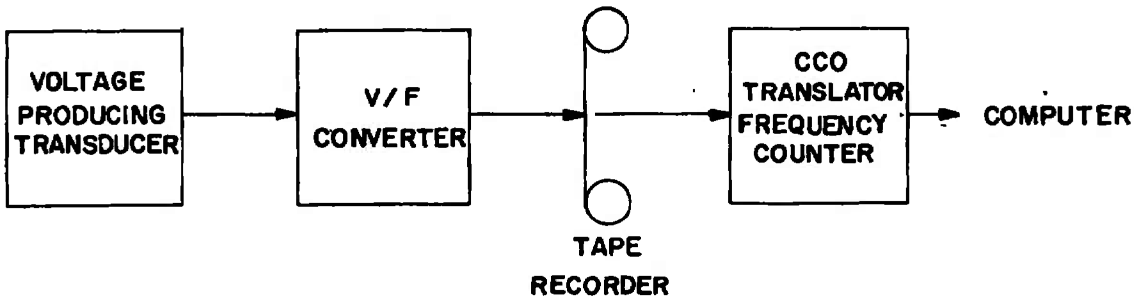


Fig. 1 Block Diagram of LRF V/F Acquistition and Recovery System

The V/F converter develops output pulses at a rate precisely proportional to the input voltage. The output pulses are recorded on magnetic tape for later data reduction. The recorded signals are recovered by means of a translator, which works like a gated frequency counter that counts the pulses during a time interval, τ. The counts contained in a summing interval are converted to a representative voltage, $V_{\tau n}$, by a digital computer using the calibration of the V/F converter and summing interval of the translator.

To illustrate: Let the V/F converter be adjusted so that the output is N_o counts in τ seconds with the input shorted. Then apply a calibration voltage of V_c volts to the input of the V/F converter; let N_c be the counts in τ seconds of the output. For a number of counts in the nth interval of τ for an arbitrary input of $V_i(t)$ defined as $N_{\tau n}$, the following expression defines $V_{\tau n}$:

$$V_{\tau_n} = \frac{N_{\tau_n} - N_o}{N_c - N_o} V_c \qquad (1)$$

But, because the V/F data recovery technique is an integrating method,

$$N_{\tau_n} - N_o \equiv K_v \int_{n\tau}^{(n+1)\tau} V_i(t)\, dt \qquad (2a)$$

and

$$N_c - N_o \equiv \int_0^{\tau} K_v V_c\, dt = K_v V_c \tau \qquad (2b)$$

where K_V is the gain of V/F converter with units of pulses per second-volt.

Substituting the definitions of Eqs. (2a) and (2b) into Eq. (1), it becomes:

$$V_{\tau_n} = \frac{1}{\tau}\int_{n\tau}^{(n+1)\tau} V_i(t)\,dt \tag{3}$$

Equation (3) will be used to theoretically simulate the V/F conversion method of data acquisition and analyze the errors resulting from taking the value of V_{τ_n} as a data point that occurs at the midpoint of the summing interval from $n\tau$ to $(n+1)\tau$ and the points linearly connected to represent the input waveform. Figure 2 illustrates this type of data acquisition and waveform reconstruction.

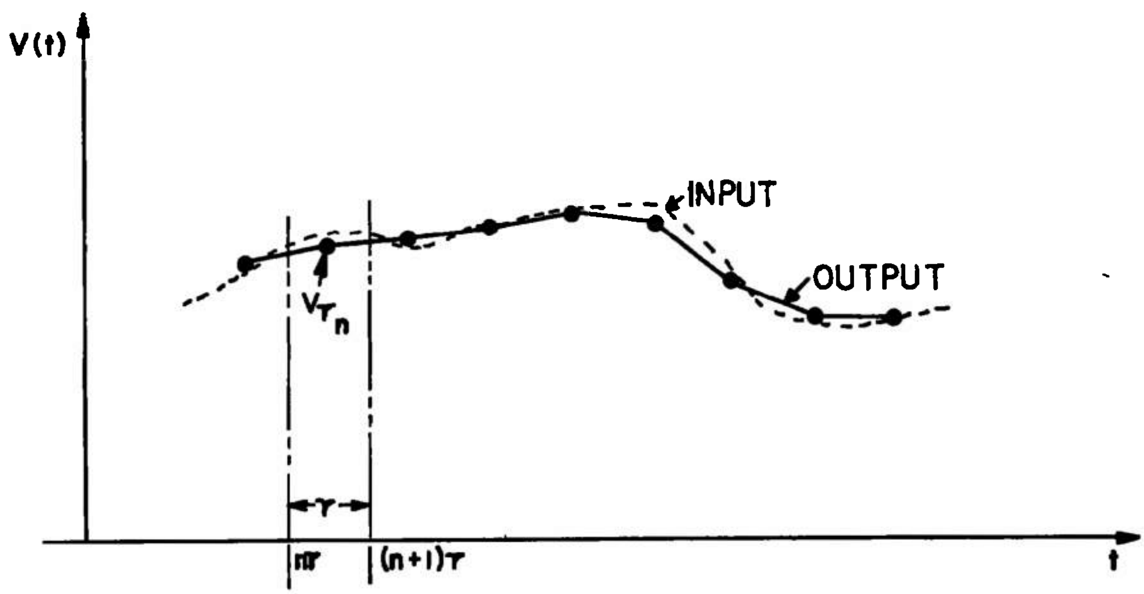


Fig. 2 Example Illustrating the Summing Interval Method of Data Representation

Since the value V_{τ_n} is an integral, its value and the value of $V_i(t)$ at $t = \tau_n = \frac{2n+1}{2}\tau$, the time at the midpoint of the nth interval, are not always equal; therefore, the error analysis also investigates the data point error. To distinguish the two, the lower case letter, e, designates the error from linear interpolation and the upper case letter, E, is used to designate the error at data points.

SECTION III
SINUSOIDAL INPUT

For the sinusoidal analysis, let $V_i(t) = \sin \frac{2\pi t}{T}$, which is normalized for ease of application and does not affect the actual error analysis.

Substituting this function in Eq. (3), the expression for the representative output becomes:

$$V_{\tau_n} = \frac{1}{\tau} \int_{n\tau}^{(n+1)\tau} \sin \frac{2\pi t}{T} \, dt$$

$$= - \frac{T}{2\pi\tau} \left[\cos \frac{2\pi t}{T} \right]_{n\tau}^{(n+1)\tau}$$

$$= - \frac{T}{2\pi\tau} \left[\cos \frac{2\pi(n+1)\tau}{T} - \cos \frac{2\pi n\tau}{T} \right] \tag{4}$$

Using the identity from Ref. 3, page 18 for the difference of two cosine functions, Eq. (4) becomes:

$$V_{\tau_n} = \frac{T}{\pi\tau} \left[\sin \frac{(2n+1)\pi\tau}{T} \cdot \sin \frac{\pi\tau}{T} \right] \tag{5}$$

Letting the ratio T/τ, which is the number of summing intervals per period, be designated with the letter m, the expression in Eq. (5) becomes:

$$V_{\tau_n} = \frac{m}{\pi} \left[\sin \frac{(2n-1)\pi}{m} \cdot \sin \frac{\pi}{m} \right] \tag{6}$$

3.1 LINEAR INTERPOLATION ERROR ANALYSIS OF SINUSOIDAL INPUT

An expression for e consists of n equations; therefore, the determination of a maximum value is a tremendous task to perform manually. Consequently, a digital computer program was written to determine the errors for a sinusoidal input as m is varied from 1 to 25.0 in increments of 0.1. The program was designed to search the number of cycles that include a whole number of summing intervals. The reason for this approach to the e_{max} determination is that, after this number of cycles, the errors become repetitive. By incorporating this test, the e_{max} determination is expedited. The program is listed in Table I-I (Appendix).

The solutions for e_{max} versus m are plotted in Fig. 3. The plot depicts an irregular relationship, and the most significant fluctuations are in the range of m = 1.0 to 3.0. Therefore, little faith can be placed in the application of the basic sampling theorem of two summing intervals per period to the V/F system. Another significant factor is that, when the sampling rate is close to frequency components of the data that have prominent magnitude, the errors greatly reduce the accuracy of the data recovery.

As an example of the use of Fig. 3, consider an input voltage signal that contains a 50-Hz component. What error would be introduced in the data reduction of this component if τ were 10 msec? What is the error for τ = 2 msec? For τ = 10 msec,

$$m = \frac{10^3}{(50)(10)} = 2$$

From Fig. 3, e_{max} = 44 percent.

For τ = 2 msec,

$$m = \frac{10^3}{(50)(2)} = 10$$

From Fig. 3, e_{max} = 6 percent.

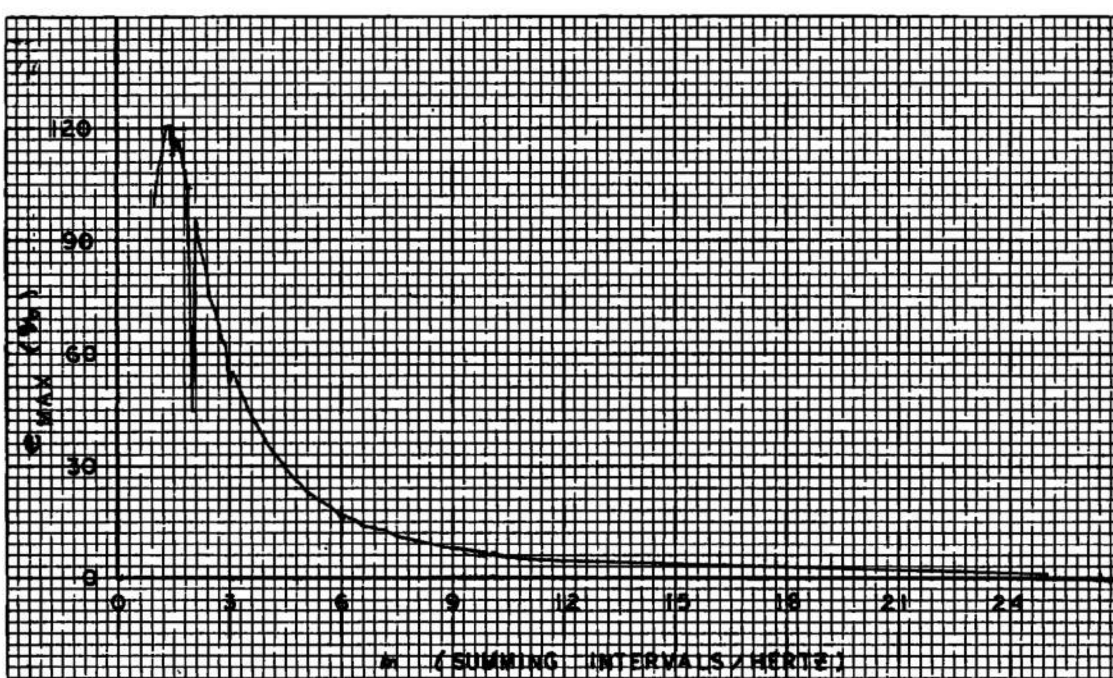


Fig. 3 Plot of m versus e_{max} for Sinusoidal Input

Figure 4 depicts the input-output relationship for these two summing intervals for the 50-Hz component.

In order that e_{max} be 5 percent or less for a sinusoidal input, m must be greater than 11.4 and at m = 2, e_{max} = 46 percent. This indicates a large error at the rate given by the sampling theorem.

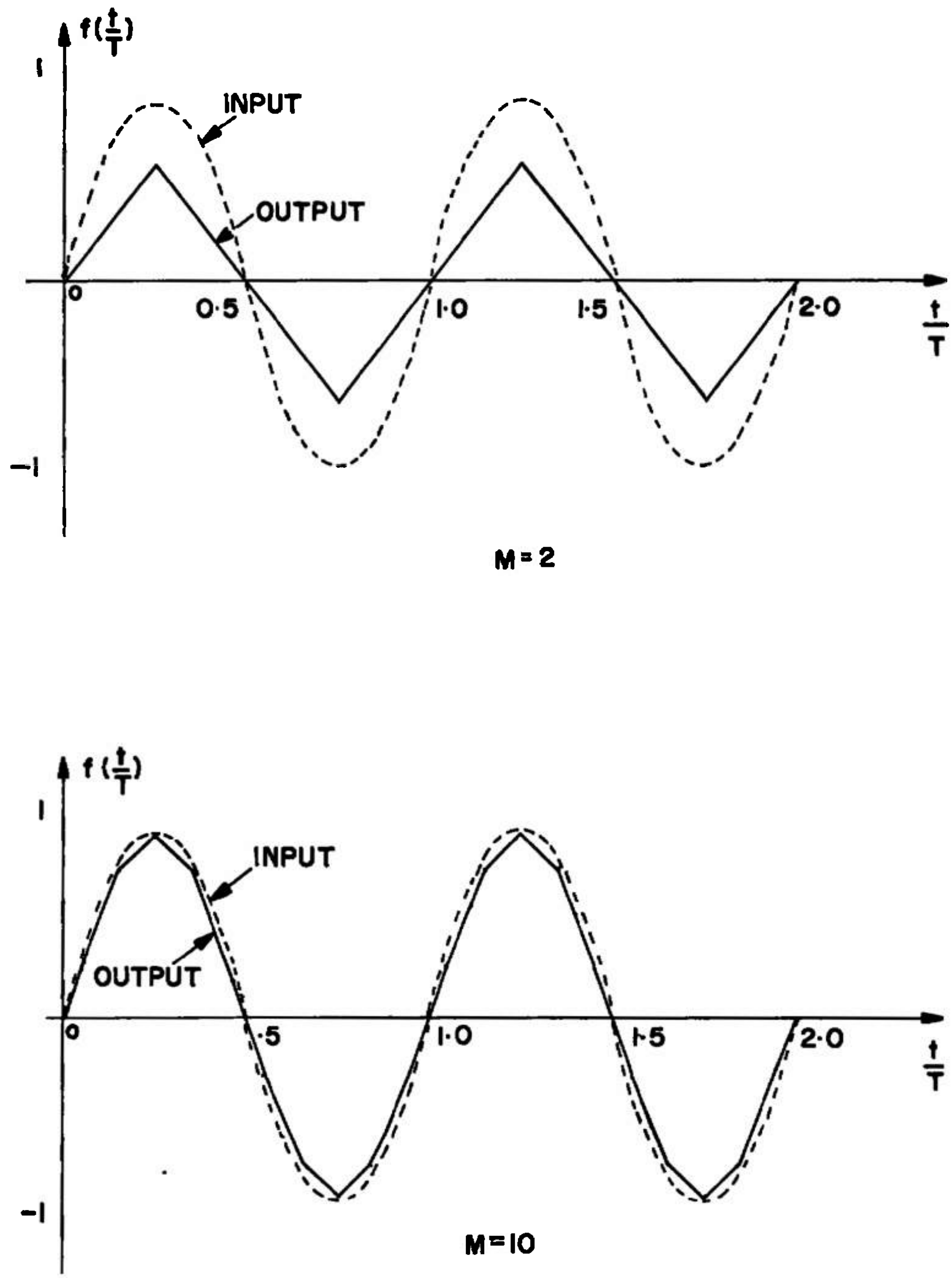


Fig. 4 Input-Output Waveforms for Sample Problem

3.2 DATA POINT ERROR ANALYSIS OF SINUSOIDAL INPUT

The error analysis for E_{max} is much simpler than for e_{max}. The data point error is defined by this relationship:

$$E = 100\,|V_i(\tau_n) - V_{\tau_n}| \tag{7}$$

since $|V_i(t)|_{max}$ is equal to one.

Combining Eqs. (6) and (7) and substituting the normalized sinusoid with the definition of m included, the expression for E becomes:

$$E = 100\,\left|\sin \frac{(2n \mp 1)\pi}{m}\left(1 - \frac{m}{\pi}\sin\frac{\pi}{m}\right)\right| \tag{8}$$

Even though the error is expressible in one equation, many calculations are required to find E_{max}. Therefore, the program used to find e_{max} also has an E_{max} option. These solutions are depicted in Fig. 5.

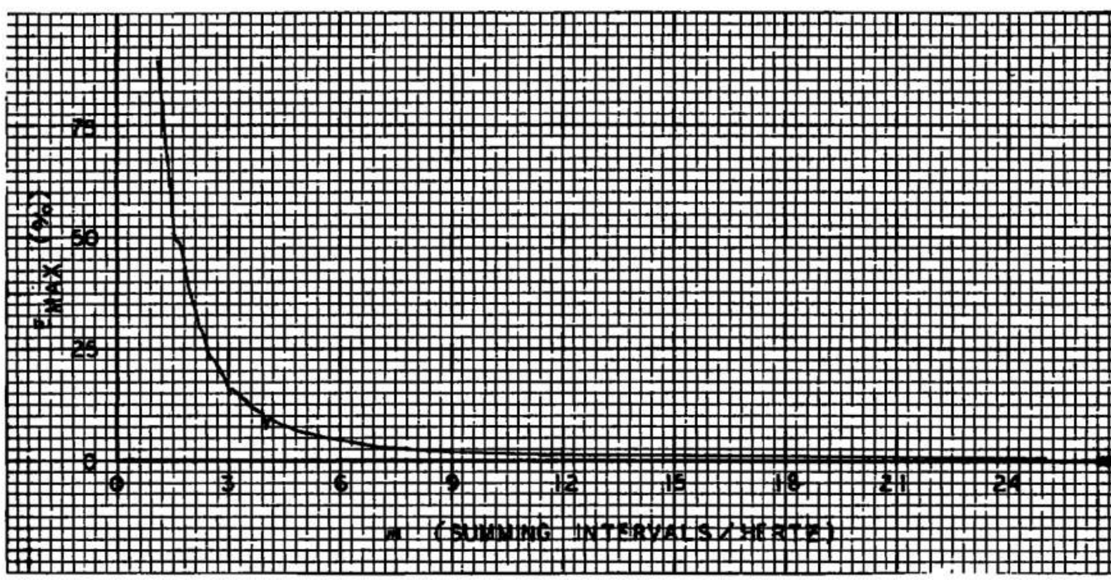


Fig. 5 Plot of m versus E_{max} for Sinusoidal Input

This plot of E_{max} versus m is a smoother curve than the one for e_{max} versus m. The minimum value of m for all data points to be within 5 percent is 5.7, and at the basic sampling rate given by the sampling theorem (two summing intervals/period), the data point is in error at a maximum by 36^+ percent.

3.3 SYNCHRONIZATION EFFECTS ON ERRORS OF SINSUOIDAL INPUT

The analyses of the sinusoid for e_{max} and E_{max} have made the assumption that the sinusoid and the n = 0 interval both started at a time when the value of the sinusoid was zero and had a positive slope. This section deals with the effects of synchronization upon e_{max} and E_{max}.

The input signal must be expressed with a synchronization parameter, ψ. The input then is:

$$V_i(t) = \sin\left(\frac{2\pi t}{T} + \psi\right)$$

Substituting this expression for $V_i(t)$ into Eq. (3):

$$V_{\tau_n} = \frac{1}{\tau}\int_{n\tau}^{(n+1)\tau} \sin\left[\frac{2\pi}{T}t + \psi\right] dt$$

$$= -\frac{T}{2\pi\tau}\cos\left[\frac{2\pi}{T}t + \psi\right]_{n\tau}^{(n+1)\tau}$$

$$= -\frac{T}{2\pi\tau}\cos\left[\frac{2\pi(n+1)\tau}{T} + \psi\right] - \cos\left[\frac{2\pi n\tau}{T} + \psi\right]$$

$$= \frac{T}{\pi\tau}\sin\left[(2n+1)\frac{\pi\tau}{T} + \psi\right]\cdot\sin\frac{\pi\tau}{T} \tag{9}$$

Using the definition of m, Eq. (9) becomes:

$$V_{\tau_n} = \frac{m}{\pi}\sin\left[\frac{(2n+1)\pi}{m} + \psi\right]\cdot\sin\frac{\pi}{m} \tag{10}$$

Since many solutions of Eq. (10) are required to determine the synchronization effects, a program was written to vary m from 1 to 25.0 in increments of 0.5 and to vary ψ from 0 to 180 deg in increments of 1 deg. The solutions for the E_{max} option are in Table I and the solutions for the e_{max} option are in Table II. The program is listed in Table I-II (Appendix).

TABLE I

E_{max} SOLUTIONS OF SYNCHRONIZATION EFFECTS ON SINUSOIDAL INPUTS

DATA POINT ERROR ANALYSIS

M	PSI	LARGEST E(MAX)	PSI	SMALLEST E(MAX)	DELTA E(MAX)
1.00	90.00	100.00	0.0	0.00	100.00
1.50	30.00	58.65	0.0	50.79	7.86
2.00	0.0	36.34	90.00	0.00	36.34
2.50	18.00	24.32	72.00	23.13	1.19
3.00	30.00	17.30	0.0	14.98	2.32
3.50	90.00	12.90	0.0	12.57	0.32
4.00	45.00	9.97	0.0	7.05	2.92
4.50	10.00	7.93	0.0	7.81	0.12
5.00	18.00	6.45	0.0	6.14	0.32
5.50	41.00	5.35	0.0	5.30	0.05
6.00	0.0	4.51	30.00	3.90	0.60
6.50	90.00	3.85	0.0	3.82	0.03
7.00	90.00	3.32	0.0	3.24	0.08
7.50	6.00	2.90	60.00	2.88	0.02
8.00	22.00	2.55	90.00	2.36	0.19
8.50	16.00	2.26	180.00	2.25	0.01
9.00	10.00	2.02	0.0	1.99	0.03
9.50	52.00	1.81	0.0	1.81	0.01
10.00	0.0	1.64	18.00	1.56	0.08
10.50	73.00	1.49	120.00	1.48	0.00
11.00	8.00	1.35	180.00	1.34	0.01
11.50	43.00	1.24	0.0	1.24	0.00
12.00	15.00	1.14	0.0	1.10	0.04
12.50	11.00	1.05	0.0	1.05	0.00
13.00	76.00	0.97	0.0	0.96	0.01
13.50	10.00	0.90	0.0	0.90	0.00
14.00	103.00	0.84	90.00	0.82	0.02
14.50	34.00	0.78	0.0	0.78	0.00
15.00	6.00	0.73	0.0	0.73	0.00
15.50	32.00	0.68	180.00	0.68	0.00
16.00	11.00	0.64	45.00	0.63	0.01
16.50	101.00	0.60	0.0	0.60	0.00
17.00	16.00	0.57	0.0	0.57	0.00
17.50	85.00	0.54	108.00	0.54	0.00
18.00	0.0	0.51	10.00	0.50	0.01
18.50	17.00	0.48	107.00	0.48	0.00
19.00	71.00	0.46	180.00	0.45	0.00
19.50	67.00	0.43	60.00	0.43	0.00
20.00	9.00	0.41	0.0	0.41	0.01
20.50	11.00	0.39	180.00	0.39	0.00
21.00	4.00	0.37	180.00	0.37	0.00
21.50	23.00	0.36	21.00	0.36	0.00
22.00	33.00	0.34	90.00	0.34	0.00
22.50	2.00	0.32	12.00	0.32	0.00
23.00	59.00	0.31	0.0	0.31	0.00
23.50	6.00	0.30	46.00	0.30	0.00
24.00	8.00	0.29	180.00	0.28	0.00
24.50	46.00	0.27	136.00	0.27	0.00
25.00	4.00	0.26	108.00	0.26	0.00

TABLE II

e_{max} SOLUTIONS OF SYNCHRONIZATION EFFECTS ON SINUSOIDAL INPUTS

M	PSI	LARGEST E(MAX)	PSI	SMALLEST E(MAX)	DELTA E(MAX)
1.00	0.0	100.00	9.00	99.95	0.05
1.50	90.00	120.67	0.0	113.77	6.91
2.00	90.00	100.00	0.0	44.65	55.35
2.50	90.00	76.61	0.0	73.76	2.85
3.00	90.00	58.65	0.0	52.15	6.50
3.50	90.00	45.69	0.0	44.68	1.02
4.00	90.00	36.34	45.00	26.89	9.45
4.50	90.00	29.47	0.0	29.03	0.44
5.00	162.00	24.32	36.00	23.18	1.14
5.50	90.00	20.38	164.00	20.15	0.23
6.00	90.00	17.30	0.0	15.06	2.24
6.50	90.00	14.86	55.00	14.74	0.12
7.00	90.00	12.90	0.0	12.56	0.33
7.50	90.00	11.29	0.0	11.20	0.10
8.00	90.00	9.97	68.00	9.24	0.73
8.50	90.00	8.86	0.0	8.79	0.07
9.00	90.00	7.93	0.0	7.79	0.14
9.50	90.00	7.13	126.00	7.08	0.05
10.00	126.00	6.45	0.0	6.14	0.32
10.50	90.00	5.86	123.00	5.81	0.05
11.00	90.00	5.35	100.00	5.27	0.08
11.50	90.00	4.90	69.00	4.85	0.05
12.00	0.0	4.51	135.00	4.32	0.19
12.50	54.00	4.16	108.00	4.15	0.01
13.00	90.00	3.85	127.00	3.79	0.06
13.50	90.00	3.57	23.00	3.53	0.04
14.00	90.00	3.32	0.0	3.19	0.13
14.50	90.00	3.10	57.00	3.07	0.03
15.00	18.00	2.90	36.00	2.86	0.04
15.50	90.00	2.72	54.00	2.70	0.02
16.00	0.0	2.55	55.00	2.47	0.08
16.50	90.00	2.40	19.00	2.38	0.02
17.00	90.00	2.26	0.0	2.21	0.05
17.50	90.00	2.13	108.00	2.12	0.02
18.00	90.00	2.02	104.00	1.97	0.05
18.50	90.00	1.91	0.0	1.90	0.02
19.00	90.00	1.81	79.00	1.79	0.03
19.50	90.00	1.72	50.00	1.71	0.02
20.00	0.0	1.64	117.00	1.62	0.02
20.50	90.00	1.56	20.00	1.54	0.02
21.00	90.00	1.49	33.00	1.46	0.02
21.50	90.00	1.42	26.00	1.39	0.02
22.00	90.00	1.35	105.00	1.31	0.04
22.50	90.00	1.29	180.00	1.26	0.03
23.00	90.00	1.24	23.00	1.20	0.03
23.50	90.00	1.19	0.0	1.15	0.04
24.00	0.0	1.14	43.00	1.09	0.05
24.50	90.00	1.09	0.0	1.04	0.05
25.00	11.00	1.05	108.00	1.05	0.00

These solutions depict the cases in which the variations of E_{max} or e_{max} are the largest. The cases are the integer values of m and m with a decimal part of 0.5. All other values of m exhibit less variation in E_{max} or e_{max} as ψ varies.

From Table I,

$$\text{for } m > 6,\ \Delta E_{max} \leq 0.19 \text{ percent}$$

$$\text{for } m > 11.4,\ \Delta E_{max} \leq 0.04 \text{ percent}$$

$$\text{for } m \geq 14.0,\ \Delta E_{max} \leq 0.01 \text{ percent}$$

The m > 11.4 values yield accurate recovery.

From Table II,

$$\text{for } m > 8,\ \Delta e_{max} \leq 0.32 \text{ percent}$$

$$\text{for } m > 11.4,\ \Delta e_{max} \leq 0.19 \text{ percent}$$

$$\text{for } m > 16,\ \Delta e_{max} \leq 0.05 \text{ percent}$$

This clearly indicates that synchronization has negligible effect on the error at a summing interval-to-period ratio that is required to accurately recover the data.

3.4 EXPERIMENTAL CORRELATION OF THEORETICAL ERRORS

A verification, by actual errors encountered in the data recovery, of the theoretical treatment was desired; therefore, an experiment to determine actual errors was formulated. For the input waveform, a sinusoid was chosen. There were, basically, two reasons for this choice: (1) a sinusoid is easily obtainable from a sinusoidal generator, and (2) the frequency or period of cycle is a very important factor in the error of a signal.

The VIDAR Data Acquisition System was set up with a sinusoidal generator connected to the input of a channel of the V/F converter. The frequency output was recorded, using direct electronics, on a magnetic tape recorder. The recorded tape was translated using a summing interval of 2 msec, and the raw counts were printed out.

In order to get the negative values of the sinusoid, the VIDAR was adjusted so that for zero input, the output counts were at midscale.

Then a known voltage of 1.5 v was applied, and the VIDAR spanned to nearly full scale. The count levels of these adjustments were:

$$0\text{v Input} = 25,003 \text{ counts/sec}$$

$$1.5\text{v Input} = 47,500 \text{ counts/sec}$$

The signal from the sinusoid generator was set at 20 Hz and 2 v peak-to-peak which is easily accommodated in magnitude by the chosen calibration voltage.

Since the theoretical error analysis used m as the variable, m had to be determined for this case of the translation summing interval; m was defined as T/τ or $1/f\tau$; giving m = 1/20 (0.002) = 25.

Synchronization of output and input for actual error analysis proved to be a difficult problem; therefore, since for an m of 25, e_{max} = 1.05 percent and E_{max} = 0.26 percent, this linear interpolated curve was used as the reference or standard. Small m's were obtained by taking two or more adjacent summing intervals at a time. A computer program, depicted in Table I-III (Appendix), was written to do the task of summing up adjacent summing intervals. The program limits the number of adjacent summing intervals to 13 since this gives an m of 1.92, which is below the level set forth by the sampling theorem. Table III is the solution for this program which has the option of determining either e_{max} or E_{max}.

As shown in Table III, the maximum errors determined by the theoretical approach were confirmed to be reasonably accurate; therefore, the general rule of thumb of 11.4 summing intervals per period for $e_{max} \leq 5$ percent is a valid one. This also indicates that the theoretical simulation is a reasonably accurate error determination technique.

SECTION IV
STEP INPUT

In the theoretical analysis of errors of the V/F converter data acquisition system in following a step input, the time reference of t = 0 is set at the start of the summing interval in which the step function occurs.

TABLE III

e_{max} and E_{max} THEORETICAL AND ACTUAL SOLUTIONS FOR SINUSOIDAL INPUT

M	ACTUAL E(MAX)	THEORY E(MAX)	M	ACTUAL E(MAX)	THEORY E(MAX)
1.92	103.31	103.68	2.08	94.82	95.68
2.27	85.46	86.51	2.50	75.63	73.76
2.78	64.50	65.81	3.13	54.01	54.39
3.57	43.32	44.08	4.17	33.76	33.75
5.00	23.88	23.18	6.25	16.34	15.98
8.33	10.47	9.19	12.50	5.35	4.15

DATA POINT ERROR ANALYSIS

M	ACTUAL E(MAX)	THEORY E(MAX)	M	ACTUAL E(MAX)	THEORY E(MAX)
1.92	38.67	38.83	2.08	32.38	33.75
2.27	28.57	28.88	2.50	22.85	23.13
2.78	20.31	19.96	3.13	15.35	15.98
3.57	13.47	12.38	4.17	8.57	9.19
5.00	8.00	6.14	6.25	4.29	4.15
8.33	4.76	2.35	12.50	0.01	1.05

The input waveform, a normalized step function, is denoted by the symbol:

$$\mu_1\ (t - a) = \begin{cases} 0 & t < a \\ 1 & t \geq a \end{cases} \tag{11}$$

For this analysis, the input is:

$$V_i(t) = \mu_1\ (t - K\tau) \tag{12}$$

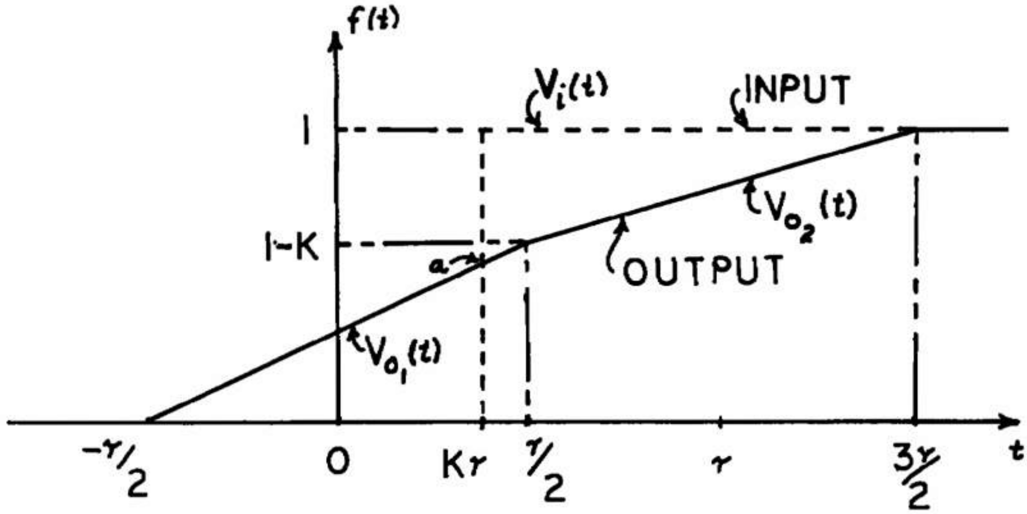
where $0 \leq K \leq 1$ is the shift of the step with respect to the summing interval reference.

Using Eqs. (3) and (12) to find the output, the particular interval for n = 0 is the only one of interest since in all intervals for n < 0, $V_{\tau_n} = 0$, and in all intervals for n > 0, $V_{\tau_n} = 1$.

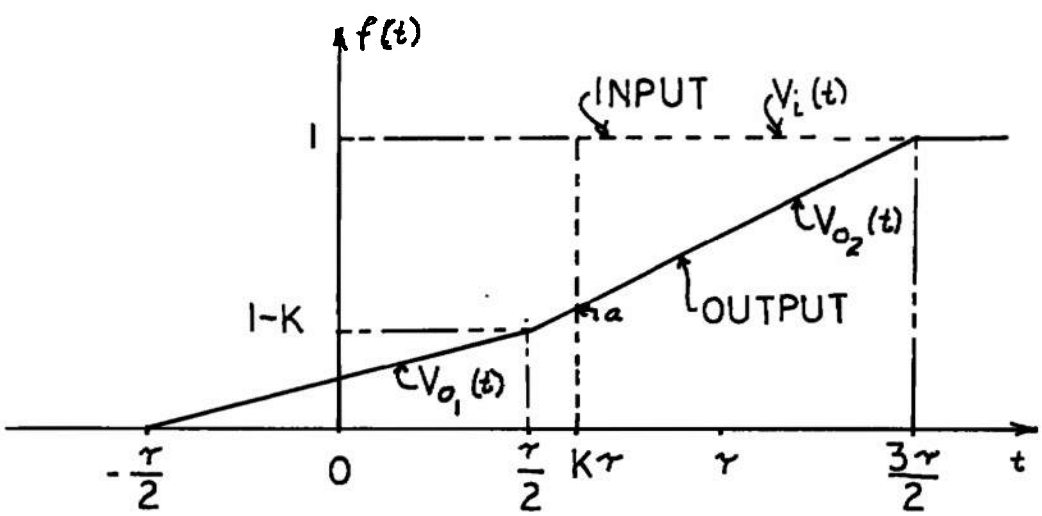
The interval for n = 0 has an output value of:

$$\begin{aligned} V_{\tau_0} &= \frac{1}{\tau}\int_0^\tau \mu_1\ (t - K\tau)\, dt \\ &= \frac{1}{\tau}\int_{K\tau}^\tau dt = \frac{1}{\tau}\left(\tau - K\right) \\ &= 1 - K \end{aligned} \tag{13}$$

The input-output relationship is depicted in Fig. 6.



$0 \leq K \leq 1/2$



$1/2 \leq K \leq 1$

Fig. 6 Input-Output Waveforms for Step Input

4.1 LINEAR INTERPOLATION ERROR ANALYSIS OF STEP INPUT

As shown in Fig. 6, the maximum error occurs at $t = K\tau$. Point a is used to designate this intersection of the input and output waveforms at $t = K\tau$. The output line from $t = -\frac{\tau}{2}$ to $\frac{\tau}{2}$ is designated $V_{o_1}(t)$, and the output line from $t = \frac{\tau}{2}$ to $\frac{3\tau}{2}$ is designated $V_{o_2}(t)$. For $0 \leq K \leq 1/2$, point a occurs on the output line $V_{o_1}(t)$. The equation for this line, which has a slope of $\frac{1}{\tau}(1-K)$ and passes through the point $(-\frac{\tau}{2}, 0)$, is:

$$V_{o_1}(t) = \frac{1}{\tau}(1 - K)(t + \frac{\tau}{2})$$

Since $|V_1(t)|_{max} = 1$,

$$\begin{aligned} e_{max} &= 100\, V_{o_1}(K\tau) \\ &= 100\,(1 - K)(K + \tfrac{1}{2}) \\ &= 50\,(1 - K - 2K^2) \end{aligned} \tag{14}$$

for $0 \leq K \leq 1/2$.

For $1/2 \leq K \leq 1$, point a occurs on the output line $V_{o_2}(t)$. The equation for this line, which has a slope of K/τ and passes through the point $(\frac{3\tau}{2}, 1)$, is:

$$V_{o_2}(t) = \frac{K}{\tau}(t - \frac{3\tau}{2}) + 1$$

Likewise,

$$\begin{aligned} e_{max} &= 100\,[1 - V_{o_2}(K\tau)] \\ &= -100K\,(K - 3/2) \\ &= 150K - 100K^2 \\ &= 50\,(3K - 2K^2) \end{aligned} \tag{15}$$

for $1/2 \leq K \leq 1$.

Figure 7 is a plot of e_{max} versus K using Eqs. (14) and (15). As can be seen, e_{max} varies from the lowest value of 50 percent at K = 0, 1/2, and 1 to a highest value of 56.25 percent at K = 1/4 and 3/4.

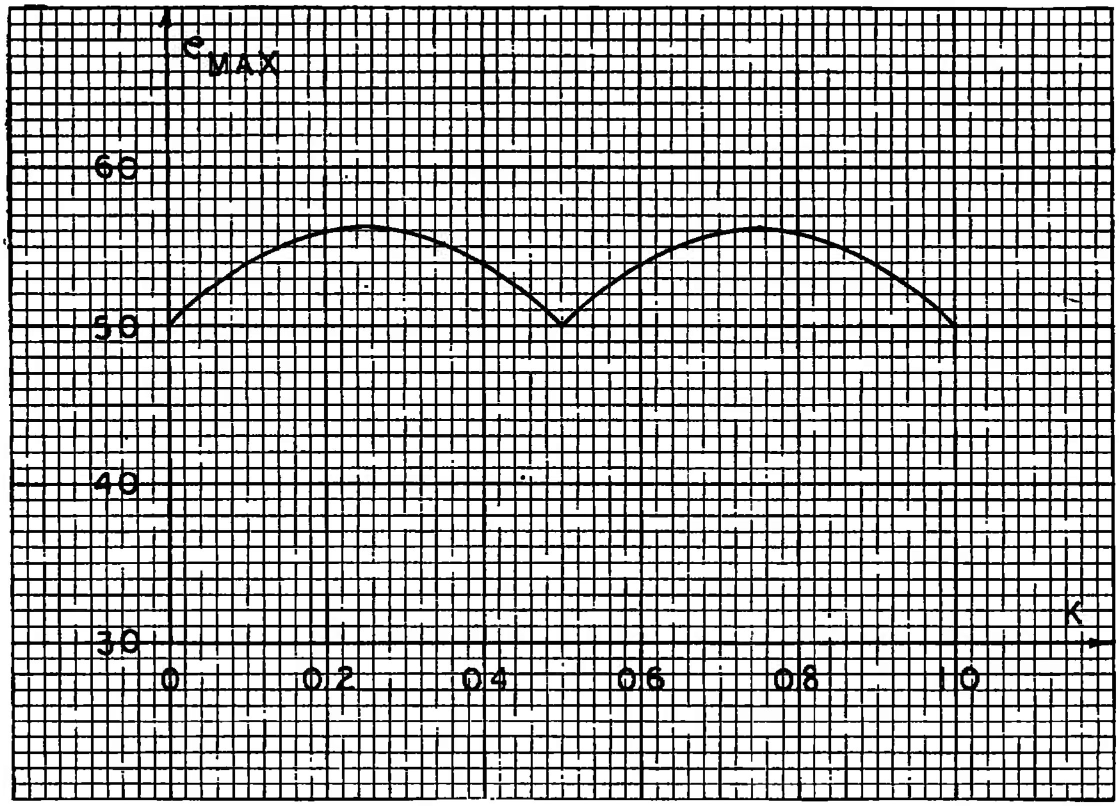


Fig. 7 Plot of K versus e_{max} for Step Input

4.2 DATA POINT ERROR ANALYSIS OF STEP INPUT

For this input signal, all data point errors are 0 percent with the exception of the one for n = 0. This data point of interest occurs at time $t = \tau_o$ or 0.5.

Using Eq. (13) and the fact that $|v_i(t)|_{max} = 1$, the E_{max} for step input is:

$$E_{max} = 100\ |V_i(\tau_o) - (1 - K)| \tag{16}$$

However, $V_i(\tau_o)$ is dependent on the value of K in the following manner:

$$V_i(\tau_o) = \begin{cases} 1 & 0 \leq K \leq 0.5 \\ 0 & 0.5 \leq K \leq 1.0 \end{cases} \tag{17}$$

Combining Eqs. (16) and (17), the maximum error becomes:

$$E_{max} = \begin{cases} 100K & 0 \leq K \leq 0.5 \\ 100\ (1 - K) & 0.5 \leq K \leq 1.0 \end{cases}$$

A plot of E_{max} versus K is shown in Fig. 8. As was the case in the total error, the maximum data point error is independent of the summing interval length and is a function of K alone.

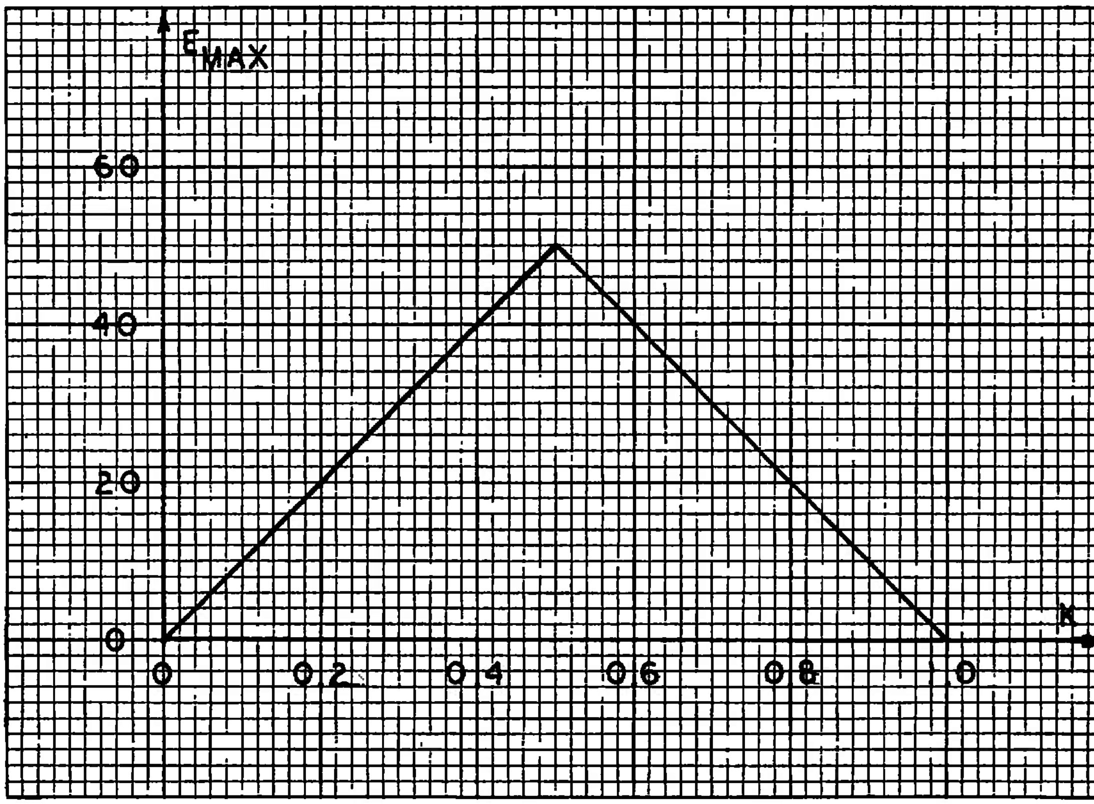


Fig. 8 Plot of K versus E_{max} for Step Input

4.3 TIME DELAY ANALYSIS OF STEP INPUT

The determination of time delay, i.e., time for the response of the linear interpolated output to be within 5 percent error, was desired. The notation T_D is used for this parameter.

This determination must, like the error analysis, be divided into two parts. Likewise, the first part uses the output, V_{o1} (t), in its determination. The ranges of K, however, are not the same. In using V_{o1} (t), the following inequality must be satisfied:

$$1 - V_{o_1}\left(\frac{T}{2}\right) \leq 0.05 \tag{18}$$

Since V_{o1} $(\frac{T}{2})$ = 1-K, Eq. (18) defines the limits of K to be:

$$0 \leq K \leq 0.05$$

For a time delay of T_D, the actual time $t = T_D + K\tau$, and from the equation for V_{o1} (t),

$$V_{o_1}(T_D + K_\tau) = 0.95 = \frac{1}{\tau}(1 - K)\left(T_D + K\tau + \frac{\tau}{2}\right)$$

Solving for T_D,

$$T_D = \frac{0.45 - 0.5K + K^2}{1 - K} \tag{19}$$

for $0 \leq K \leq 0.05$.

The second part, $0.05 \leq K \leq 1$, uses V_{o2} (t) in the determination of T_D, with $t = T_D + K\tau$.

$$V_{o_2}(T_D + K\tau) = 0.95 = \frac{K}{\tau}(T_D + K\tau - 1.5\tau) + 1$$

Solving for T_D,

$$T_D = \frac{-0.05 + 1.5K - K^2}{K^2} \tag{20}$$

for $0.05 \leq K \leq 1$.

Using Eqs. (19) and (20), a plot of T_D versus K is depicted in Fig. 9. As shown in this plot, T_D varies from a minimum of 0.449τ to a maximum of 1.053τ.

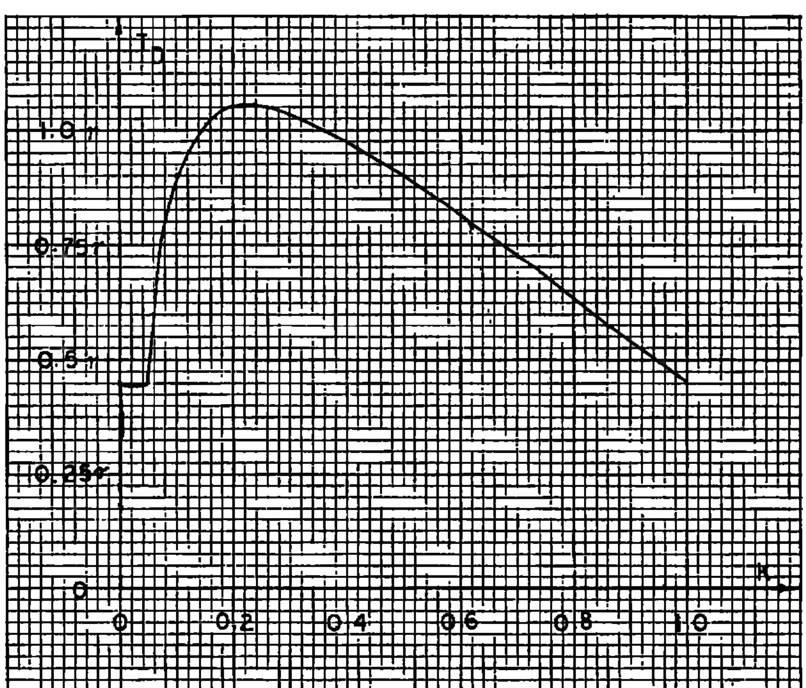


Fig. 9 Plot of K versus T_D for Step Input

The step response will always appear to be a ramp. For K = 1/2, the rise time of the ramp is 2τ sec, and for K = 0 or 1, the rise time will be τ sec; therefore, to minimize this effect, τ must be made as small as possible. Also, the output will respond to within 5 percent error in a delay time, T_D, of approximately τ.

SECTION V
RAMP INPUT

In order to theoretically analyze the error of the V/F converter data acquisition system in the data recovery of a ramp input, time, t = 0, is set at the start of the summing interval in which the ramp function begins.

The ramp function is defined as:

$$\mu_2 \, (t - a) = \begin{cases} 0 & t < a \\ t - a & t \geq a \end{cases} \tag{21}$$

For a normalized ramp function which has a rise time of β summing intervals, the input function is expressed as:

$$V_i(t) = \frac{1}{\beta\tau} \left[\mu_2 \, (t - K\tau) - \mu_2 \, (t - (K + \beta) \, \tau) \right] \tag{22}$$

The value of V_{τ_O} is dependent upon the value of K + β. For $(K + \beta) \leq 1$, the combination of Eqs. (3) and (22) becomes:

$$V_{\tau_o} = \frac{1}{\beta\tau^2} \int_0^{\tau} \left[\mu_2 \, (t - K\tau) - \mu_2 \, (t - (K + \beta) \, \tau) \right] dt$$

$$= \frac{1}{\beta\tau^2} \int_{K\tau}^{(K+\beta)\tau} (t - K\tau) \, dt + \frac{1}{\tau} \int_{(K+\beta)\tau}^{\tau} dt$$

$$= \frac{1}{\beta\tau^2} \left[\tfrac{1}{2} t^2 - K\tau t \right]_{K\tau}^{(K+\beta)\tau} + \frac{1}{\tau} \left[t \right]_{(K+\beta)\tau}^{\tau}$$

$$= \frac{1}{\beta\tau^2} \left[\tfrac{1}{2} (K + \beta)^2 \tau^2 - \tfrac{1}{2} K^2 \tau^2 - K \, (K + \beta) \, \tau^2 + K^2 \tau^2 \right]$$

$$+ \frac{1}{\tau} \left[\tau - (K + \beta) \, \tau \right]$$

$$= \frac{\beta}{2} + 1 - (K - \beta)$$

$$= 1 - K - \frac{\beta}{2} \tag{23}$$

for $0 \leq K + \beta \leq 1$.

For $(K + \beta) > 1$, V_{τ_0} is defined by the integral:

$$V_{\tau_0} = \frac{1}{\beta\tau^2}\int_{K\tau}^{\tau} (t - K\tau)\,dt$$

$$= \frac{1}{\beta\tau^2}\left[\tfrac{1}{2}t^2 - K\tau t\right]_{K\tau}^{\tau} = \frac{1}{\beta\tau^2}\left[\tfrac{1}{2} - \tfrac{1}{2}K^2 - K + K^2\right]$$

$$= \frac{(1 - K)^2}{2\beta} \tag{24}$$

for $(K + \beta) > 1$.

For all summing intervals up to, but not including, the one in which the ramp breaks to a zero slope, the value of the output is equal to the value of the input at the midpoint of the summing interval. That is:

$$V_{\tau_n} = V_i\left(\frac{2n + 1}{2}\,\tau\right)$$

for $0 < n < \mathrm{Int}\,(K + \beta)$.

For the interval $n = \mathrm{Int}\,(K + \beta)$, the output value is:

$$V_{\tau_n} = \frac{1}{\beta\tau^2}\int_{n\tau}^{(K+\beta)\tau} (t - K\tau)\,dt + \frac{1}{\tau}\int_{(K+\beta)\tau}^{(n-1)\tau} dt$$

$$= \frac{1}{\beta\tau^2}\left[\tfrac{1}{2}t^2 - K\tau t\right]_{n\tau}^{(K+\beta)\tau} + \frac{1}{\tau}\left[t\right]_{(K+\beta)\tau}^{(n+1)\tau}$$

$$= \tfrac{1}{2}(K + \beta)^2 - \frac{K(K + \beta)}{\beta} - \frac{n^2}{2\beta} + \frac{Kn}{\beta} + (n - 1) - (K + \beta)$$

$$= (n + 1) - K - \beta/2 - \frac{(n - K)^2}{2\beta} \tag{25}$$

Figure 10 depicts the input-output relationships for a ramp function.

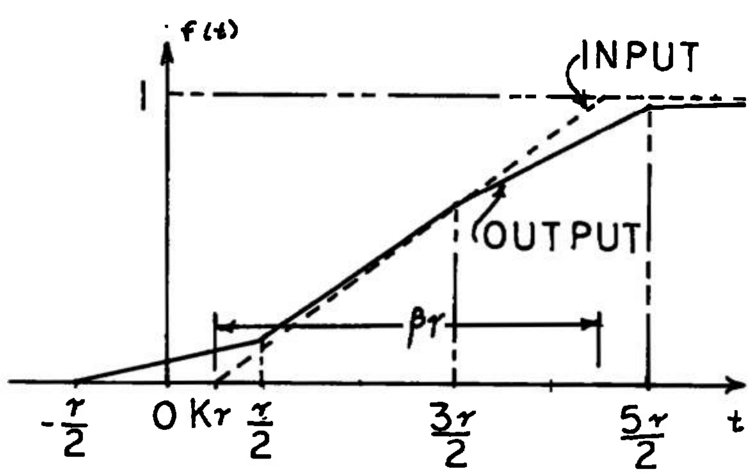


Fig. 10 Input-Output Waveforms for Ramp Input

5.1 LINEAR INTERPOLATION ERROR ANALYSIS OF RAMP INPUT

As shown in Fig. 10, the e_{max} occurs at either $t = K\tau$ or $t = (K + \beta)\tau$; therefore, e_{max} is equal to the maximum of these two values. An expression for the e_{max} is so cumbersome to deal with manually that a digital computer program, depicted in Table I-IV (Appendix), was written to calculate the many solutions required to determine the minimum β for 5 percent or less error. The program calculates the β_{min} for $e_{max} \leq 5$ percent; then β is varied from 0.1 to β_{min} in increments of 0.1 for K = 0.0 and 0.50 and the value of e_{max} determined for each set of parameters. The family of curves that was calculated is depicted in Fig. 11. These two values of K are the extreme cases; all other values are in between. The plots in Fig. 11 indicate that a value of $\beta \geq 5.0$ assures a response within 5 percent.

Fig. 11 Plot of e_{max} versus β for Ramp Input with K Constant

5.2 DATA POINT ERROR ANALYSIS OF RAMP INPUT

The value of E_{max}, as shown in Fig. 10, occurs at either $t_1 = 1/2\tau$ or $t_2 = [\text{Int}(K + \beta) + 1/2]\,\tau$. The expression for E_{max} is the maximum of:

$$E_1 = 100\,|V_i(\tfrac{1}{2}\tau) - V_{\tau_0}|$$

or

$$E_2 = 100\,|V_i\left(\frac{2n+1}{2}\tau\right) - V_{\tau_n}'$$

where n = Int (K + β).

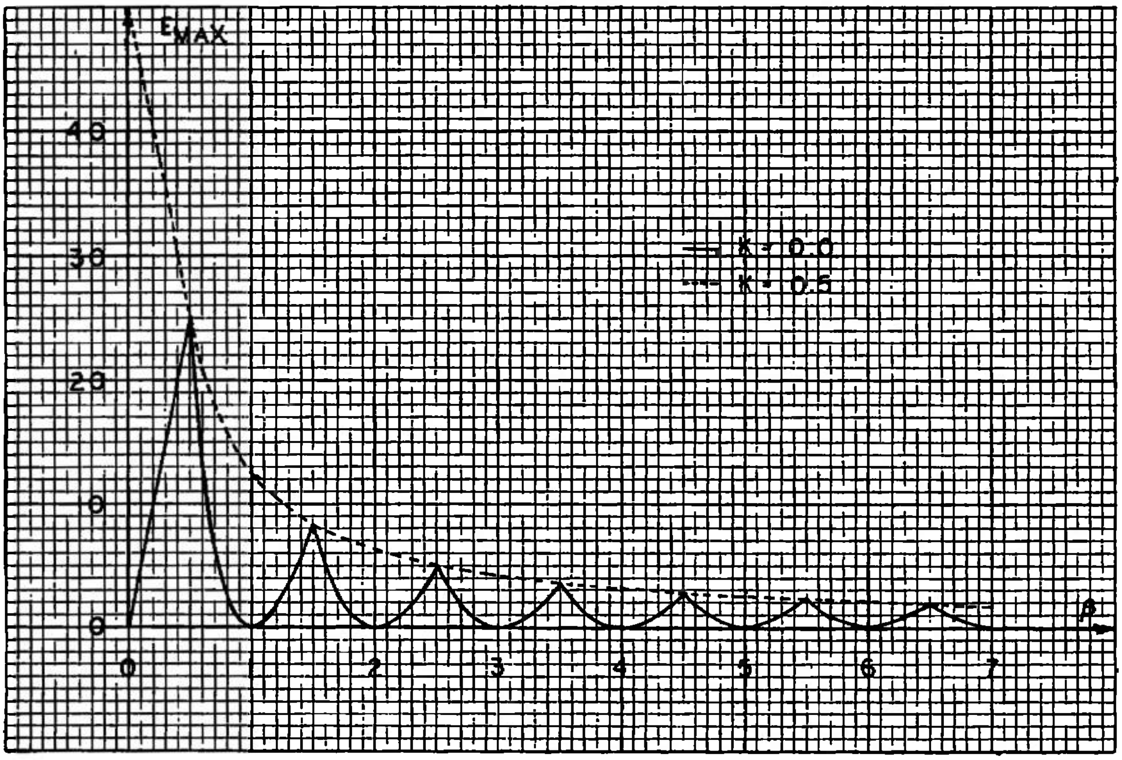


Fig. 12 Plot of E_{max} versus β for Ramp Input with K Constant

The digital computer program that calculates e_{max} was written for the E_{max} option also. The value of β_{min} for $E_{max} \leq 5$ percent is first calculated; then β is varied from 0.1 to β_{min} in increments of 0.1 for K = 0.0 and 0.5, since these errors are the extreme cases, and the value of E_{max} determined for each set of parameters. Figure 12 depicts the family of curves that was calculated. These plots indicate that, for a value of $\beta \geq 2.5$, the data points are in error by 5 percent or less. To illustrate, for a summing interval of 2 msec, any ramp with a rise time of 5 msec or greater will be reconstructed with all data points within 5 percent error.

SECTION VI
SUMMARY AND CONCLUSIONS

6.1 SUMMARY

A voltage-to-frequency data acquisition system was theoretically analyzed for errors that occur when it is used as a sampling system. Since the standard sampling theorem is not directly applicable, the

system was simulated on a digital computer to aid in the determination of parameter criteria for expected levels of accuracy for sinusoidal, step, and ramp inputs.

Parameter limits were determined for such parameters as m, summing intervals per period of a sinusoid, K, delay of step and ramp inputs with respect to start of summing interval, and β, ratio of ramp rise time to summing interval length.

An investigation of the effect of signal synchronization was accomplished for the sinusoid, and an experiment was performed, using a sinusoid, to determine the validity of the theoretical solutions.

6.2 CONCLUSIONS

The voltage-to-frequency conversion for data acquisition does not offer accurate recovery (5 percent error) of sinusoidal data until the ratio of summing interval to period is about 11.4, i.e., 11.4 summing intervals per period. This means that for a 2-msec summing interval, any frequency component above 43.9 Hz has a maximum error greater than 5 percent and, from Fig. 3, any component of 151.5 Hz or greater is in error at its maximum by more than 50 percent.

The voltage-to-frequency conversion system has an inherent error because all data points do not coincide with the input waveform. For the sinusoid, the ratio of summing intervals to period must be 12.8 for the data point to be within 1 percent of the input waveform. At the value of m = 11.4, e_{max} was 4.95 percent and E_{max} is 1.26 percent; therefore, a sizeable portion of the error is the result of the data point error.

The step input error is independent of the summing interval and is symmetrical with respect to K = 0.5 for both e_{max} and E_{max}. However, delay time, time required to bring the output to be within 5 percent of the input, is a function of K and τ, and as K varies, the time delay varies in the range from 0.45τ to 1.05τ.

Ramp inputs can be followed within 5 percent if the ratio of ramp rise time to summing interval is larger than five. This is to say that a 2-msec summing interval can accurately follow a ramp of rise time greater than 10 msec.

The ramp function has an unusual E_{max} versus β for constant K relationship. At low and high values of K, the E_{max} oscillates as a function of β. At midvalues of K, the relationship is nearly exponential.

If $\beta \geq 12.5$, all data points are within 1 percent of the input waveform. The reverse conditions exist in the relationship of e_{max} versus β for constant K.

An additional error that may be significant at small summing interval lengths is resolution error from the low pulse counts that occur. However, errors from resolution were not included in this analysis.

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2. Tucker, G. K., and Willis, D. M. A Simplified Technique of Control System Engineering, Minneapolis-Honeywell Regulator Company, Brown Instrument Division, Philadelphia, 1962.

3. Burington, R. S., Compiler. Handbook of Mathematical Tables and Formulas. Handbook Publishers, Inc., Sandusky, Ohio, 1954.

APPENDIX
COMPUTER PROGRAMS

TABLE I-1

COMPUTER PROGRAM TO FIND E_{max} AND e_{max} FOR A SINUSOIDAL INPUT AS m VARIES FROM 1.0 TO 25.0 IN INCREMENTS OF 0.1

```
/JOB GO,TIME=10
            DIMENSION XMA(241),DIFMA(241)
C           IND=0  (E(MAX) FOR DATA POINT)
C           IND=1  (E(MAX) FOR LINEAR INTERPOLATION AT ANY POINT)
            READ(5,1) IND
1           FORMAT(I10)
            PSI=0.
            DO 103 IM=10,250
            I=IM-9
            XM=FLOAT(IM)/10.
            XMA(I)=XM
            DIFMA(I)=VEMAX(XM,PSI,IND)
103         CONTINUE
            WRITE(6,2)
2           FORMAT('1')
            IF(IND)100,100,101
100         WRITE(6,3)
3           FORMAT(10X'DATA POINT ERROR ANALYSIS'/)
101         WRITE(6,4) (XMA(J),DIFMA(J),J=1,241)
4           FORMAT(4(5X'M'4X'E(MAX)'4X)//4(2F8.2,4X))
            STOP
            END
```

TABLE I-1 (Concluded)

```
      FUNCTION VEMAX(XM,PSI,IND)
      DIMENSION FA(252),TA(252)
      PI=3.1415926
      PSI=PSI*PI/180.
      IF(XM-25.)99,99,98
98    STOP 2525
99    DO 110 L=10,100,10
      XP=0.001
      DO 109 I=1,2
      IXC=XM*FLOAT(L)+XP
      IF(IXC-(IXC/10)*10)108,108,110
108   XP=XP+0.9
109   CONTINUE
      GO TO 111
110   CONTINUE
111   NS=(IXC+25)/10
      ARG1=PI/XM
      EMAX=0.
      DO 100 N=1,NS
      XN=N-2
      ALPHA=2.*XN+1.
      TA(N)=ALPHA/XM/2.
      ARG2=ALPHA*ARG1+PSI
      FA(N)=SIN(ARG2)*SIN(ARG1)/ARG1
      ERR=ABS(SIN(ARG2)-FA(N))
      EMAX=AMAX1(ERR,EMAX)
100   CONTINUE
      IF(IND)103,103,101
101   EMAX=0.
      IXS=TA(NS)*100.+1.
      DO 102 IX=1,IXS
      X=FLOAT(IX)/100.
      TEST=TAB2(FA,TA,X,NS)
      DEV=ABS(SIN(2.*PI*X+PSI)-TEST)
      EMAX=AMAX1(EMAX,DEV)
102   CONTINUE
103   VEMAX=EMAX*100.
      RETURN
      END
```

TABLE I-II
COMPUTER PROGRAM TO FIND THE LARGEST, SMALLEST, AND DELTA E_{max} OR e_{max} as ψ VARIES FROM 0 TO 180 deg IN INCREMENTS OF 1 deg AND m VARIES FROM 1 TO 25 IN INCREMENTS OF 0.5

```
      DIMENSION TA(252),FA(252)
      PI=3.1415926
      DO 906 ND=1,2
      IND=ND-1
      WRITE(6,1)
    1 FORMAT('1')
      IF(IND)902,902,903
  902 WRITE(6,2)
    2 FORMAT(//15X'DATA POINT ERROR ANALYSIS')
  903 WRITE(6,3)
    3 FORMAT(/23X'LARGEST'13X'SMALLEST    DELTA'/7X
     1'M'2X2(6X'PSI'5X'E(MAX)'),4X'E(MAX)')
      DO 906 IM=10,250,5
      XM=FLOAT(IM)/10.
      DO 99 I=1,252
   99 TA(I)=(2.*FLOAT(I-2)+1.)/XM/2.
      EMAOUT=0.
      EMIOUT=150.
      DO 102 L=10,100,10
      XP=0.001
      DO 101 I=1,2
      IXC=XM*FLOAT(L)+XP
      IF(IXC-(IXC/10)*10)100,100,102
  100 XP=XP+0.9
  101 CONTINUE
      GO TO 103
  102 CONTINUE
  103 NS=(IXC+25)/10
      ARG1=PI/XM
      DO 900 I=1,181
      CSI=I-1
      PSI=CSI*PI/180.
      EMAX=0.
      DO 104 N=1,NS
      ARG2=(2.*FLOAT(N-2)+1.)*ARG1+PSI
      FA(N)=SIN(ARG2)*SIN(ARG1)/ARG1
      ERR=ABS(SIN(ARG2)-FA(N))*100.
      EMAX=AMAX1(ERR,EMAX)
  104 CONTINUE
      IF(IND)107,107,105
  105 EMAX=0.
      IXS=TA(NS)*100.
      K=2
      DO 106 IX=1,IXS
      X=FLOAT(IX-1)/100.
  701 DIF=X-TA(K)
      IF(DIF)703,703,702
  702 K=K+1
      GO TO 701
  703 DEV=ABS(SIN(2.*PI*X+PSI)-DIF*(FA(K)-FA(K-1))
     1/(TA(K)-TA(K-1))-FA(K))*100.
      EMAX=AMAX1(DEV,EMAX)
```

TABLE I-II (Concluded)

```
  106 CONTINUE
  107 IF(EMAOUT-EMAX)800,801,801
  800 EMAOUT=EMAX
      PSIL=CSI
  801 IF(EMAX-EMIOUT)802,900,900
  802 EMIOUT=EMAX
      PSIS=CSI
  900 CONTINUE
      DIFMA=EMAOUT-EMIOUT
      WRITE(6,4) XM,PSIL,EMAOUT,PSIS,EMIOUT,DIFMA
    4 FORMAT(6F10.2)
  906 CONTINUE
      STOP
      END
```

TABLE I-III
COMPUTER PROGRAM TO COMPARE THEORETICAL AND ACTUAL e_{max} or E_{max} FOR SINUSOIDAL INPUT

```
/JOB GO,TIME=10
      DIMENSION DATA(292),FA(292),TA(292),FB(140),TB(140),
     1XM(12),EMAX(12),ETMAX(12)
      READ(5,1) IND,IZC,IUC,VU,DATA
1     FORMAT(3I10,F10.3/(16F5.1))
      WRITE(6,2)
2     FORMAT('1')
      TAU=0.002
      VKT=TAU*FLOAT(IUC-IZC)/VU
      ZF=FLOAT(IZC)*TAU
      VMAX=0.
      DO 100 N=1,292
      TA(N)=FLOAT(2*N-27)*TAU/2.
      FA(N)=(DATA(N)-ZF)/VKT
100   VMAX=AMAX1(VMAX,ABS(FA(N)))
      DO 104 J=2,13
      XJ=15-J
      TAUJ=TAU*XJ
      VKTJ=VKT*XJ
      TZCJ=ZF*XJ
      XM(J-1)=0.05/TAUJ
      ETMAX(J-1)=VEMAX(XM(J-1),0.,IND)
      ERR=0.
      K=279/(15-J)+1
      DO 102 L=1,K
      DN=0.
      LL=14+(15-J)*(L-2)
      LU=LL+14-J
      DO 101 I=LL,LU
101   DN=DN+DATA(I)
      FB(L)=(DN-TZCJ)/VKTJ
      TB(L)=FLOAT(2*L-3)*TAUJ/2.
      VV1=TAB2(FA,TA,TB(L),292)
      ERR=AMAX1(ABS(VV1-FB(L)),ERR)
102   CONTINUE
      IF(IND)105,105,106
106   ERR=0.
      DO 103 I=1,1001
      TIME=FLOAT(I-1)*0.0005
      VV1=TAB2(FA,TA,TIME,292)
      VV2=TAB2(FB,TB,TIME,K)
103   ERR=AMAX1(ABS(VV1-VV2),ERR)
105   EMAX(J-1)=ERR/VMAX*100.
104   CONTINUE
      IF(IND)107,107,108
107   WRITE(6,3)
3     FORMAT(10X'DATA POINT ERROR ANALYSIS')
108   WRITE(6,4) (XM(J),EMAX(J),ETMAX(J),J=1,12)
4     FORMAT(2(20X'ACTUAL'2X'THEORY'4X)/2(15X'M'4X'E(MAX)'2X
     1'E(MAX)'4X)//2(10X3F8.2,4X))
      STOP
      END
```

TABLE I-IV
COMPUTER PROGRAM TO FIND e_{max} OR E_{max} FOR A CONSTANT K AS β VARIES FROM 0.1 TO 12.0 IN INCREMENTS OF 0.1

```
/JOB GO
          DIMENSION ERR(150),B(150)
C         IND=0  (E(MAX) FOR DATA POINT)
C         IND=1  (E(MAX) FOR LINEAR INTERPOLATION AT ANY POINT)
          DO 105 J=1,2
          IND=2-J
          WRITE(6,2)
2         FORMAT('1')
          IF(IND)97,97,98
97        IP=2101362752
          GO TO 99
98        IP=1077952576
99        NB=120
          DO 105 I=1,4
          XK=FLOAT(I-1)/4.
          DO 104 IB=1,NB
          B(IB)=FLOAT(IB)*0.1
          ERR(IB)=EMAXR(B(IB),XK,IND)
104       CONTINUE
          WRITE(6,3) IP,XK,(B(IB),ERR(IB),IB=1,NB)
3         FORMAT(//' ERROR ANALYSIS FOR BETA'A1' WITH K = 'F5.2
         1//6(4X'BETA  E(MAX)'4X)//6(2F8.2,4X))
105       CONTINUE
          STOP
          END
          FUNCTION EMAXR(BETA,XK,IND)
          DIMENSION FN(23),TN(23)
C         IND=0  (E(MAX) FOR DATA POINT)
C         IND=1  (E(MAX) FOR LINEAR INTERPOLATION AT ANY POINT)
          RAMPF(T)=AMIN1(1.,AMAX1(T-XK,0.)/BETA)
          FN(1)=0.0
          TN(1)=-0.5
          DO 100 J=2,23
          TN(J)=TN(J-1)+1.
100       FN(J)=1.0
          TS1=XK+BETA
          IS1=TS1
          TS2=IS1
          IS2=IS1+1
          TS3=TS2-XK
          FN(2)=1.-XK-BETA/2.
          IF(IS1-1)105,101,101
101       FN(2)=(1.-XK)**2/BETA/2.
          IF(IS1-2)104,102,102
102       DO 103 J=3,IS2
103       FN(J)=RAMPF(TN(J))
104       FN(IS2+1)=TS3+1.-(BETA+TS3**2/BETA)/2.
105       IF(IND)106,106,107
106       EM1=ABS(RAMPF(0.5)-FN(2))
          EM2=ABS(RAMPF(TS2+0.5)-FN(IS2+1))
          GO TO 108
107       EM1=ABS(TAB2(FN,TN,XK,13))
          EM2=ABS(TAB2(FN,TN,TS1,13)-1.)
108       EMAXR=AMAX1(EM1,EM2)*100.
          RETURN
          END
```

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13. ABSTRACT

The errors generated by the recovery of data acquired by a voltage-to-frequency (V/F) data acquisition system are theoretically analyzed, using a digital computer. The input waveforms considered are the sinusoid, the step function, and the ramp function. In each case, the well known "sampling theorem" of C. E. Shannon, viz, two samples per period of the smallest period present in the signal are sufficient for recovery of data with perfect fidelity, will be shown to be inapplicable as a criterion for accurate and reliable recovery of data acquired by a V/F system. The errors are of two categories: One is the error from linear interpolation recovery, and the other is from the fact that data points do not always coincide with the input.

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2. Data -- acquisition

3 Error analysis